A Discontinuous Galerkin Method for the M_1 Model of Radiative Transfer

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Summary

- A model for energy transfer via photon radiation is required for several applications
- Full radiation transfer equations are too expensive to solve in most cases
- Research has been devoted to finding simpler models
- The M_1 model is one such simplified model which has been recently studied
- We developed a first order implicit-explicit (IMEX) scheme to solve the model by discontinuous Galerkin (DG) methods
- We tested the model with the shadow cone problem [1]

Background

- M_1 model is a simplified model which preserves properties of full radiative transfer equations
- E – Energy, F – Flux, T – Material Temperature, P – Radiative Pressure, σ_a – Emission Cross Section, σ_t – Total Cross Section, a – Radiation Constant , c – Speed of Light, c_v – Heat Capacity

$$\partial_t E + \nabla_x \cdot F = -c\sigma_a(E - aT^4)$$

$$\partial_t F + c^2 \nabla_x \cdot P(E,F) = -c\sigma_t F$$

$$\partial_t T = \frac{c\sigma_a}{c_v}(E - aT^4)$$

- Challenge is to ensure solution satisfies |F| ≤ cE, since particles can't move faster than light
- Model is ill-posed when condition is violated

Numerical Method

- IMEX Scheme: source terms treated implicitly (Backward Euler); flux terms treated explicitly (Forward Euler)
- Implicit step performed on point-wise values; values mapped to coefficients in polynomial basis
- DG Formulation (in two dimensions) to update coefficients:

$$\iint_{I_k} \partial_t E^h \, v \, \mathrm{d}x\mathrm{d}y = \iint F^h \, \partial_x v \, \mathrm{d}x\mathrm{d}y - \int F^\star v \mathrm{d}y\Big|_{x_{k+\frac{1}{2}}} + \int F^\star v \mathrm{d}y\Big|_{x_{k-\frac{1}{2}}} + \iint F^h \, \partial_y v \, \mathrm{d}y\mathrm{d}x - \int F^\star v \mathrm{d}x\Big|_{y_{k+\frac{1}{2}}} + \int F^\star v \mathrm{d}x\Big|_{y_{k-\frac{1}{2}}}$$

$$\iint_{I_k} \partial_t F^h \, v \, \mathrm{d}x\mathrm{d}y = \iint P^h \, \partial_x v \, \mathrm{d}x\mathrm{d}y - \int P^\star v \mathrm{d}y\Big|_{x_{k+\frac{1}{2}}} + \int P^\star v \mathrm{d}y\Big|_{x_{k-\frac{1}{2}}} + \iint P^h \, \partial_y v \, \mathrm{d}y\mathrm{d}x - \int P^\star v \mathrm{d}x\Big|_{y_{k+\frac{1}{2}}} + \int P^\star v \mathrm{d}x\Big|_{y_{k-\frac{1}{2}}}$$

Numerical Method (Cont.)

$$F^\star = \frac{1}{2}(F_n^+ + F_n^-) - \frac{c}{2}(E_n^+ - E_n^-)$$

$$P^\star = \frac{1}{2}(P_n^+ + P_n^-) - \frac{c}{2}(F_n^+ - F_n^-)$$

- v is from orthogonal polynomial basis at most degree 2
- Explicit step performed on basis coefficients
- Limiter to ensure |F|≤ cE at each time step
- Coefficients mapped to point-wise values for implicit step

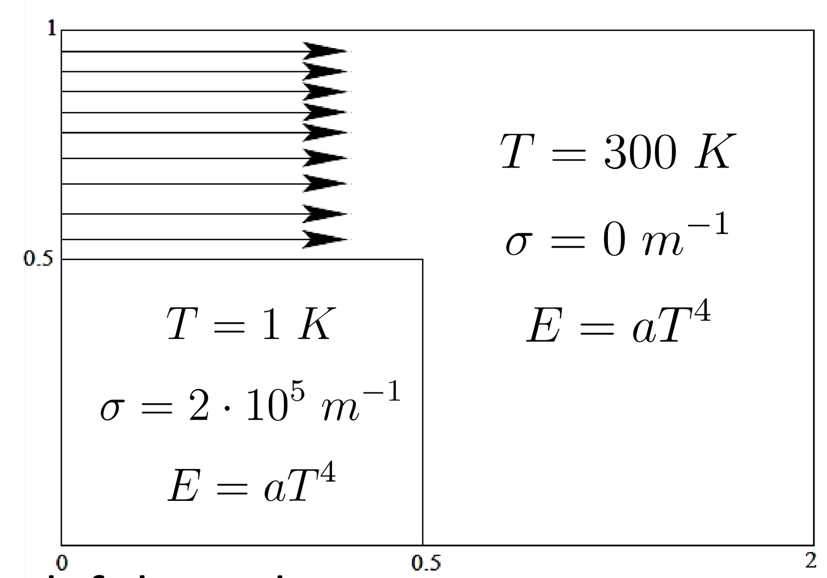
Results: 2D Shadow Cone

- Initial conditions

$$c = 3 \cdot 10^8 \ ms^{-1}$$

$$a = 1.372 \cdot 10^4 \ Jm^{-3}K^{-4}$$

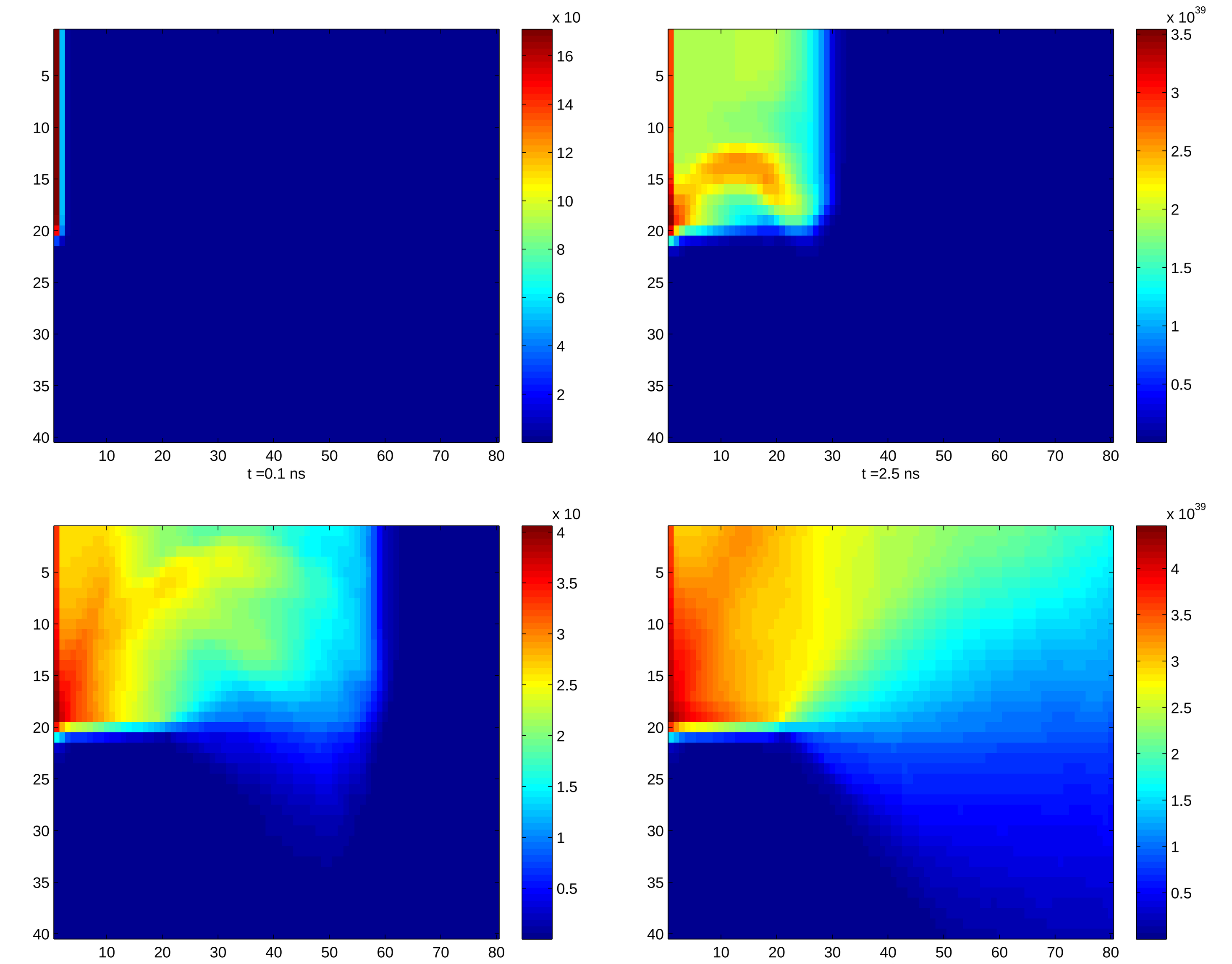
$$c_v = 8.6 \cdot 10^5 \ Jm^{-3}K^{-1}$$



- Boundary Condition in upper left boundary

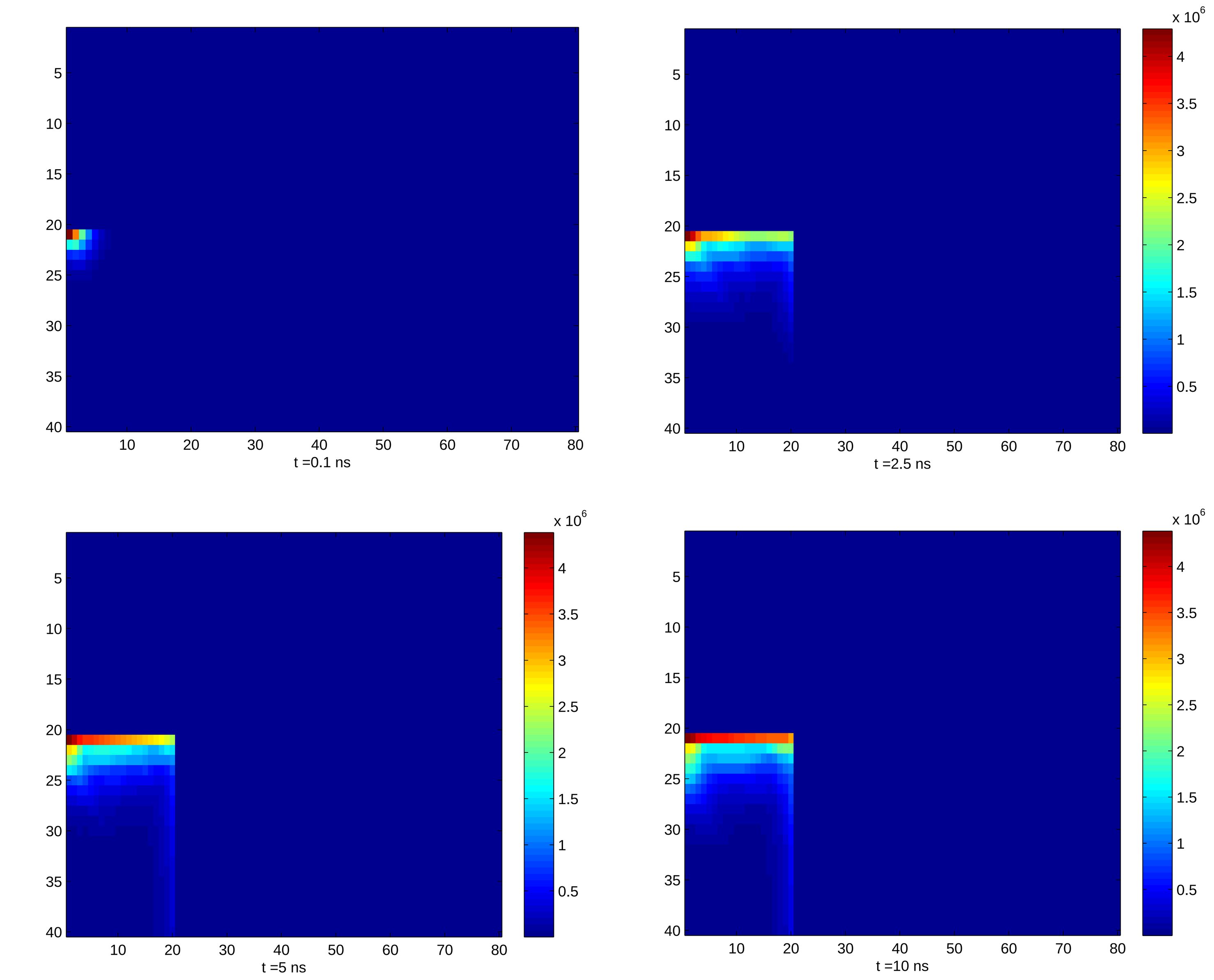
$$T_r = 5.802 \cdot 10^6 \ K \qquad E = aT_r^4$$

- Simulation run until final time of 10 ns
- Evolution of Energy (E)



Results (Cont.)

- Evolution of Material Temperature (T)



Conclusion

- Goals to implement in future work:
- Less diffusive numerical flux
- Higher order IMEX method

References

[1] Berthon, Charrier, and Dubroca. *An HLLC Scheme to Solve the M_1 Model of Radiative Transfer in Two Space Dimensions*. J SCI COMPUT (2007)

[2] Olbrant, Hauck, and Frank. *A Realizability-Preserving Discontinuous Galerkin Method for the M_1 Model of Radiative Transfer.* J COMPUT PHYS (2012)

Acknowledgements

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